Key

Math I

Name_____

5-3 Practice

Date_____

The midpoint of a segment is the point that divides the segment into two congruent pieces. The midpoint of the segment that joins points (x_1, y_1) ***and*** (x_2, y_2) is the point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

To find the midpoint of the segment joining, (x_1, y_1) ***and*** (x_2, y_2), average the two x values and average the two y values.

Find the midpoint of the segment with the following endpoints:

Example 1. $(5, 8)$ and $(2,6)$

Answer:

$$\left(\frac{5+2}{2}, \frac{8+6}{2}\right)$$

$$\left(\frac{7}{2}, \frac{14}{2}\right)$$

$$\left(\frac{7}{2}, 7\right)$$

Example 2. $(-8, 3)$ and $(4, -1)$

Answer:

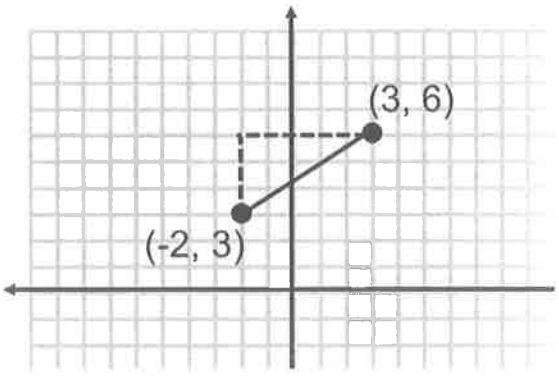
$$\left(\frac{-8+4}{2}, \frac{3+(-1)}{2}\right)$$

$$\left(\frac{-4}{2}, \frac{2}{2}\right)$$

$$(-2, 1)$$

The distance, d, between two points with coordinates (x_1, y_1) ***and*** (x_2, y_2) can be found using the Pythagorean Theorem

Example 3. Find the distance between (-2, 3) and (3, 6).



Find the vertical distance between the y-coordinates.

$\Delta y = 6 - 3 = 3$

Find the horizontal distance between the x-coordinates.

$\Delta x = 3 - (-2) = 5$

Use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to solve for the distance.

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$34 = c^2$

$\sqrt{34} = c$

Find the coordinates of the midpoint of the segment joining the given points.

1. (0, 2) and (6, 4)

$M = \left(\frac{0+6}{2}, \frac{2+4}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right) = \boxed{(3, 3)}$

2. (-11, 3) and (8,-7)

$M = \left(\frac{-11+8}{2}, \frac{3+-7}{2}\right) = \left(\frac{-3}{2}, \frac{-4}{2}\right) = \boxed{\left(-\frac{3}{2}, -2\right)}$

3. (2.3, 3.7) and (1.5, -2.9)

$M = \left(\frac{2.3+1.5}{2}, \frac{3.7+-2.9}{2}\right) = \left(\frac{3.8}{2}, \frac{0.8}{2}\right) = \boxed{(1.9, 0.4)}$

4. $(x, 2)$ and $(x + 4, -4)$

$M = \left(\frac{x+x+4}{2}, \frac{2+-4}{2}\right) = \left(\frac{2x+4}{2}, \frac{-2}{2}\right) = \boxed{(x+2, -1)}$

Find the distance between the two points.

5. (-4, 2) and (2,-1)

$6^2 + 3^2 = d^2 \rightarrow$ distance

$36 + 9 = d^2$

$\boxed{\sqrt{45} = d \approx 6.71}$

6. (-2, -3) and (-2, 4)

vertical line: x-values are the same!

$\boxed{d = 7}$

7. (3, 2) and (5, -2)

$d^2 = 2^2 + 4^2$

$\boxed{d = \sqrt{20} \approx 4.47}$

8. (5, -7) and (8,-2)

$d^2 = 3^2 + 5^2$

$\boxed{d = \sqrt{34} \approx 5.83}$

For the given endpoints of a diameter in a circle, find

a. the center of the circle (Midpoint)

b. the radius of the circle → Distance from midpoint to endpoint

either endpoint ↑

9. (-8, 6) and (0, 0)

Center = Midpoint = $\left(\frac{-8}{2}, \frac{6}{2}\right)$ = $\boxed{(-4, 3)}$

Radius: $d^2 = 4^2 + 3^2$

$d = \sqrt{25}$ = $\boxed{5 \text{ units}}$

10. (4,-9) and (-2, -9)

Center = M = $\left(\frac{4+-2}{2}, \frac{-9+-9}{2}\right)$ = $\boxed{(1, -9)}$

11. The midpoint of two coordinates is (5, 7). If one coordinate is (8, 3), what is the other coordinate? (x, y)

$(5, 7) = \left(\frac{8+x}{2}, \frac{3+y}{2}\right)$

$5 = \frac{8+x}{2}$ → $10 = 8 + x$ → $2 = x$

$7 = \frac{3+y}{2}$ → $14 = 3 + y$ → $11 = y$

$\boxed{(2, 11)}$

12. The midpoint of two coordinates is (-3, 2). If one coordinate is (4, 5), what is the other coordinate?

$(-3, 2) = \left(\frac{4+x}{2}, \frac{5+y}{2}\right)$

$\frac{4+x}{2} = -3$ → $4 + x = -6$ → $x = -10$

$\frac{5+y}{2} = 2$ → $5 + y = 4$ → $y = -1$

$\boxed{(-10, -1)}$

13. The distance between two coordinates is $\sqrt{89}$. If the coordinates are (7, 3) and (15, y), what value(s) could y be?

$(\sqrt{89})^2 = 8^2 + (y-3)^2$

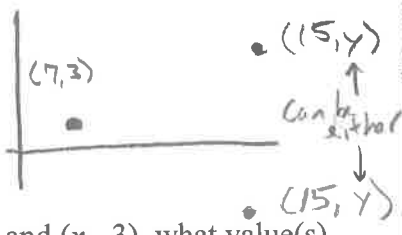
$89 = 64 + (y-3)^2$

$25 = (y-3)^2$

$\pm 5 = y - 3$

$5 = y - 3$ → $\boxed{8 = y}$

$-5 = y - 3$ → $\boxed{-2 = y}$



14. The distance between two coordinates is $\sqrt{109}$. If the coordinates are (2, 7) and (x, -3), what value(s) could x be?

$(\sqrt{109})^2 = (2-x)^2 + 10^2$

$109 = (2-x)^2 + 100$

$9 = (2-x)^2$

$\pm 3 = 2 - x$

$3 = 2 - x$ → $1 = -x$ → $\boxed{x = -1}$

$-3 = 2 - x$ → $-5 = -x$ → $\boxed{x = 5}$